Third Semester B.E. Degree Examination, Feb./Mar. 2022

Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Evaluate (i) $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$ (ii) $L(t^2 e^{-3t} \sin 2t)$ **(06 Marks)**

b. If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$, $f(t + 2a) = f(t)$ then show that $L(f(t)) = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$ **(07 Marks)**

c. Solve by using Laplace Transforms
$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t},\ y(0) = 0,\ y'(0) = 0$$ **(07 Marks)**

OR

2 a. Evaluate $L^{-1}\left(\frac{4s+5}{(s+1)^2(s+2)}\right)$ **(06 Marks)**

b. Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$ by using convolution theorem. **(07 Marks)**

c. Express $f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \sin 2t, & \pi \le t < 2\pi \\ \sin 3t, & t \ge 2\pi \end{cases}$

in terms of unit step function and hence find its Laplace Transform. **(07 Marks)**

Module-2

3 a. Obtain fourier series for the function $f(x) = |x|$ in $(-\pi, \pi)$ **(06 Marks)**

b. Expand $f(x) = \frac{(\pi - x)^2}{4}$ as a Fourier series in the interval $(0, 2\pi)$ and hence deduce that
$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots$$ **(07 Marks)**

c. Express y as a Fourier series upto the second harmonic given :

x:	0	60	120	180	240	300
y:	4	3	2	4	5	6

(07 Marks)

OR

4 a. Find the Half-Range sine series of $\pi x - x^2$ in the interval $(0, \pi)$ **(06 Marks)**

b. Obtain fourier expansion of the function $f(x) = 2x - x^2$ in the interval $(0, 3)$. **(07 Marks)**

c. Obtain the Fourier expansion of y upto the first harmonic given :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

Module-3

5 a. If $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$, find the Fourier transform of f(x) and hence find the value of $\int_0^\infty \frac{\sin x}{x} dx$ (06 Marks)

b. Find the infinite Fourier cosine transform of $e^{-\alpha x}$. (07 Marks)

c. Solve using z-transform $y_{n+2} - 4y_n = 0$ given that $y_0 = 0, y_1 = 2$ (07 Marks)

OR

6 a. Find the fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$; $m > 0$. (06 Marks)

b. Obtain the z-transform of $\cos n\theta$ and $\sin n\theta$. (07 Marks)

c. Find the inverse z-transform of
$$\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$$
(07 Marks)

Module-4

7 a. Solve $\frac{dy}{dx} = x^3 + y$, $y(1) = 1$ using Taylor's series method considering up to fourth degree terms and find y(1.1). (06 Marks)

b. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ compute y(0.2) by taking h = 0.2 using Runge – Kutta method of fourth order. (07 Marks)

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040 and y(0.3) = 2.090, find y(0.4) correct to 4 decimal places using Adams-Bashforth method. (07 Marks)

OR

8 a. Use fourth order Runge-Kutta method, to find y(0.8) with h = 0.4, given $\frac{dy}{dx} = \sqrt{x+y}$, y(0.4) = 0.41 (06 Marks)

b. Use modified Euler's method to compute y(20.2) and y(20.4) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5 Taking h = 0.2. (07 Marks)

c. Apply Milne's predictor-corrector formulae to compute y(2.0) given $\frac{dy}{dx} = \frac{x+y}{2}$ with

x	0.0	0.5	1.0	1.5
y	2.000	2.6360	3.5950	4.9680

(07 Marks)

Module-5

9 a. Using Runge-Kutta method, solve

$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$, for x = 0.2, correct to four decimal places, using initial conditions $y(0) = 1, y'(0) = 0$ **(07 Marks)**

b. Derive Euler's equation in the standard form viz, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ **(07 Marks)**

c. Find the extremal of the functional $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x)\,dx$ **(06 Marks)**

OR

10 a. Given the differential equation $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ and the following table of initial values:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y′	2	2.3178	2.6725	2.0657

Compute y(1.4) by applying Milne's Predictor-corrector formula. **(07 Marks)**

b. Prove that geodesics of a plane surface are straight lines. **(07 Marks)**

c. On what curves can the functional $\int_0^1 (y'^2 + 12xy)\,dx$ with y(0) = 0, y(1) = 1 can be extremized? **(06 Marks)**

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